Optimization of the mesh of Riemann surface
(Extended Abstract)

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The visualization of complex valued function is difficult because of the dimension of ambient space to which the graphs are naturally situated. We can identify the set of complex numbers $\mathbb{C}$ with the plane $\mathbb{R}^2$. Therefore the function $f\colon \mathbb{C} \to \mathbb{C}$ can be visualized as $f\colon \mathbb{R}^2 \to \mathbb{R}^2$. Hence, the graph of such function is situated in four-dimensional space.

Our algorithm depicts the graph of the function over the Riemann sphere in the three-dimensional Euclidean space [Jones and Singerman, 1987]. The modulus of the complex number defines the distance of the point from the unit sphere. The argument of the complex number defines the color of the point. Using this technique, we can get the approximation of the graph of the complex function over the whole domain.

In previous work, we used the approximation of the sphere by creating the polyhedron constructed by the spherical coordinates. Uneven distribution of the vertices was a disadvantage of this method [Bátorová et al., 2013]. Therefore, we examined other method – approximation of the sphere using icosahedron and its subdivision. With the use of the new approach, we obtained a smaller average error in comparison with a model having similar number of vertices used by the former approach.

The next step was to implement an adaptive grid, that provides subpartition of the grid in the areas with rapid change of the modulus and argument of the function.

The fig. 1 shows the graph of the function $f(z) = \tan(z)$ after eight iteration of the adaptive division of the mesh. The average error in the graph constructed by the spherical coordinates was 0.0161 and the mesh has 5472 vertices. The average error in the graph constructed by the icosahedron was 0.0124 and the mesh has 4114 vertices. The error is calculated after transformation by function $\arctan(f(z))$. We can see, that despite lower amount of vertices in the first case, we acquired lower average error.

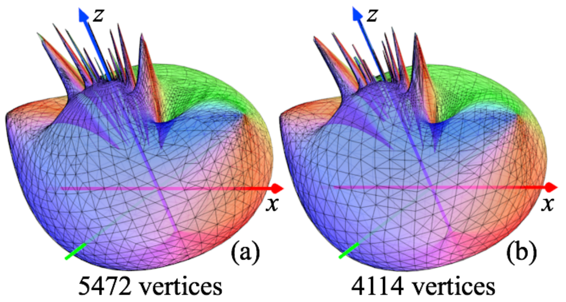

Figure 1: Model of $f(z) = \tan(z)$ constructed by (a) the icosahedron, (b) the spherical coordinates.

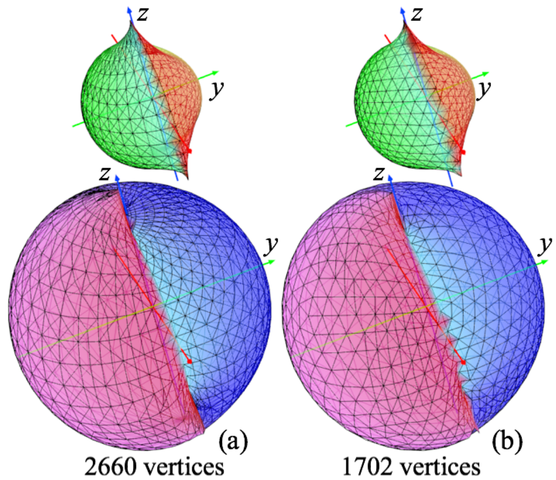

Figure 2: Model of $f(z) = \sqrt{z-0.5}$ constructed by (a) the icosahedron, (b) the spherical coordinates. Final model is made of composition of two parts.

We also used this method to construct the graphs of the multivalued functions $f(z) = \sqrt[n]{p(z)}$, where $p(z)$ is a polynomial function and $n \in \mathbb{N}$, see. fig. 2.

References

[Bátorová et al., 2013] Bátorová, M., Valíková, M., and Chalmovianský, P. (2013). Desingularization of ade singularities via deformation. Spring conference on Computer Graphics, to appear, 29.

[Jones and Singerman, 1987] Jones, G. A. and Singerman, D. (1987). Complex functions: an algebraic and geometric viewpoint. Press Syndicate of the University of Cambridge, Great Britain.

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